REVIEW

Concerning: **assoc. professor** competitionin in scientific specialty: 4.5 mathematics, announced by UF in the field of higher education 4. "Natural Sciences, Mathematics and Informatics", in professional field 4.5. "Mathematics", scientific specialty "Mathematical Analysis", in the discipline "Higher Mathematics", for the needs of the Department of Mathematics, Physics and Informatics, announced in SG, no. 102 of 07.12.2021, procedure code WWI -Asp -1121-76.

With a single candidate

Chief Assistant Dr. Yuri Mitkov Dimitrov

By: prof. George Vencislavov Boiadjiev, PhD, Scientific specialty: 4.5 mathematics, theoretical mechanics, Faculty of mathematics and informatics, Sofia University „St. Kliment Ohridski", member of the scientific jury (Order ZPS-34 / 28.01.2022 of the Rector of University of Forestry, on the basis of a decision of the FS of the Faculty of Forestry, with protocol № 25 of 25.01.2022).

The review is prepared in accordance with the requirements of:

1. Law on the Development of the Academic Staff in the Republic of Bulgaria (LASRB).
2. Regulations for application of the Law for the development of the academic staff in the Republic of Bulgaria.
3. Regulations for the structure and activity of UF.
4. Regulations on the terms and conditions for acquiring scientific degrees and holding academic positions in UF.

The structure of the review follows the instructions for preparation of reviews and opinions of the members of the scientific juries on procedures for holding academic positions and scientific degrees of the Faculty of Forestry of UF.

1. Presentation of the candidate and the attached documents to the competition.

Dr. Yuri Dimitrov graduated from the Baba Tonka Mathematical High School in Ruse in 1989. He then received a diploma of higher education from Sofia's Kliment Ohridski University in 1996, majoring in Mathematics. In the period 1993-1995 he participated in the Tempus program for student exchange at the University of East Anglia, Norwich, Great Britain, receiving additional qualifications. He holds a master's degree from the University of Michigan, Houghton, USA (1996-1998), where he holds a master's degree in mathematics. He received his doctorate in mathematics from Ohio State University, Columbus, USA, in 2006. In the period 2008-2010 he completed another master's degree - in Computer Science, again at Ohio State University, Columbus, USA. After returning to Bulgaria, he completed a full-time doctorate (2013-2016) at

the University of Ruse "Angel Kanchev", Ruse, with major subjects "Mathematics" and "Numerical Methods". Among his mathematical knowledge and achievements (including a silver medal at the Balkan Mathematical Olympiad, Split, Croatia, 1989), his skills in working with the Matlab, Mathematica and Latex software packages should also be mentioned. He speaks English and Russian.

2. The topic and description of the articles in the competition.

Dr. Yuri Dimitrov has a total of 26 publications, 4 of which were used to defend a doctoral dissertation. Of these, the publications at scientific conferences, symposia and journals in the country are 7, and abroad - 19. They are all in a foreign language. Only in journals there are 19 publications. 26 publications have been submitted for participation in the competition (one has been accepted for publication, but there is SJR (Scopus): 0.215), of which 2 have an impact factor, due to which it is very high - publications [10] with IF 2.239 and [23] with IF 5.606, and with impact rank [SJR] are 3. The individual publications are 6, which is a significant part of their total number. There are also 4 chapters from books - publications [8], [12], [16] and [24].

The theme of the competition is formulated as "Methods for approximation of fractional derivatives", as most of the presented publications are directly dedicated to this, and the rest - to their application.

The author's reference for the contribution character of the candidate's works is 6 pages.

3. Scientific, scientific-applied and applied contributions and the presence or absence of plagiarism.

Following the wording of the topic of the competition, it is good to say a few words at the beginning about the fractional derivatives themselves. They are first mentioned in a letter from Leibniz to Lopital in 1695. Although they were also considered as objects by Euler and Laplace, they were first introduced in the literature as a term by Lacroix in 1819. For fractional derivatives of functions, several types of definitions have been proposed, such as Riemann-Liouville, Grunwald-Letnikov, Weyl, Caputo, Marchaud, Riesz , which can be found in detail in the relevant publications Samko et al., 1993; Podlubny, 1999; Hilfer, 2000; Kilbas et al., 2006; Hilfer, 2008 et al.

Left and right fractional derivatives are defined, where the function $f:[a,b]\to R$ (a and b can be also $\pm\infty$) , that is differentiated, is a subintegral function in the composition of a definite integral with boundaries a and b , or non-proper (when one or both of them is $\pm\infty$). In the case of the left fractional derivative, the integral has a fixed lower bound a and variable upper limit t , and of the right - with a variable lower limit t and a fixed upper limit b . They participate in the definitions as parameters $\alpha\in[n-1,n)$, i.e. $n-1\le\alpha<n$, n - natural number, and the Gamma function. If in particular α is a natural number, then the definitions of fractional derivatives become definitions of ordinary derivatives of integer order.

Generally speaking, the meaning of fractional derivatives is as follows: the left fractional derivative is related to the "past" of the process f (its development from the moment a until the

moment in question t), and the right - with the "future" development of the process f from the moment in question t to the end b. If $u<t$, where t is the current moment, the state $f(u)$ belongs to the "past" of the process f; if $u>t$, then the state $f(u)$ belongs to the "future" of the process f. So the current state of the process f at the moment t, which process has been started at $u=a$, depends on all its previous states $f(u)$, where $u\in[a,t)$. Integral operators (fractional integrals) are also defined, where the "fractional parameter" is $\alpha>0$. It is proved that the fractional integrals are convergent for each integrable function f. For integers α, i.e. $\alpha=n$, they coincide with the ordinary n - multiple integrals (according to Cauchy's formula) for n - multiple integration. With the help of fractional integrals the fractional derivatives of order (left and right respectively) can be written as an operator for simple integer differentiation of order n, located in front of the corresponding fractional integral.

According to the topic of the competition and the presented list of papers for consideration, they can be grouped mainly into two main directions - one is approximations of the fractional derivatives of Grunwald-Letnikov and Caputo, and the other - numerical solutions of ordinary and partial linear fractional differential equations, and also of integral equations. This is the main scientific and scientific-applied contributions of the candidate, which, although synthesized in one sentence, are completely sufficient as achievements to successfully win the competition.

In the subsequent analysis, the reviewer argues with the relevant papers (numbered according to the proposed list), as some of the publications fall into more than one of the areas.

Articles belong to the first direction are [4, 5, 10, 11, 21, 22], and to the second - articles [1, 2, 3, 12, 10, 22, 26]. One proof of the accuracy of the approximations are the results for estimating the errors in the conducted numerical experiments. For example, in the numerical solution of ordinary fractional differential equations in the first-order Grunwald approximation and values of the "fractional parameter" α equal to 0.3 и 0.5 respectively, the errors are after the second decimal place. While in the Grunwald approximation of the second and third order (i.e. two-point and three-point approximations) and values of the "fractional parameter" α equal to 0.3, 0.5 and 0.9 respectively, the errors are between ten to minus four and minus eight degrees! This also applies to the three-point approximation for the Caputo derivative of order $3-\alpha$ where α is equal to 0.3 and 0.4. Further, in the numerical solution of the fractional equation of diffusion, (for α equal to 0.6) the results for the errors are of the same order. The fractional diffusion equation discussed in [3], the solution of which can be applied both to purely scientific contributions and to the scientific-applied contributions of the candidate, is one of the examples of the application and importance of fractional derivatives.

Other examples of this (given in principle for completeness and having nothing to do with the works presented in the competition) include: modeling of non-conservative forces and dissipation effects in the so-called mechanics of Lagrange and Hamilton; to derive the equations of a damped harmonic oscillator; to describe critical phenomena in nonequilibrium systems of physics and mechanics. It turns out that mathematical models using fractional derivatives are more adequate than "integer" models in areas not only such as classical mechanics, but mainly in

fractal dynamics, dispersion and turbulence, astrophysics, potential theory, viscoelasticity, electrodynamics, optics and thermodynamics.

Furthermore, in addition to the Caputo fractional derivative approximation, asymptotic decompositions of the trapezoid approximation are derived in [11] and [26], using Fourier transform, generating functions and the Euler-McLaren formula.

[9] derives quadratic formulas of the second order with generating functions (in particular "sequence" and "tangent") and their asymptotic decompositions of the fourth order, comparing the accuracy of these quadrature formulas with the accuracy of the quadrature formula of rectangles, and in [26] - approximations for the fractional integral of order α, $1+\alpha$, $2+\alpha$, $3+\alpha$, $4+\alpha$; (it is good to remember here that the "fractional parameter" is $\alpha > 0$).

In the context of the topic of the competition, the other articles submitted by the candidate for participation in it can be referred to the scientific-applied and applied contributions. In [6] the Black-Scholz fractional model for European options is considered, constructing three-point fourth-order approximations on an uneven network and three-point compact differential schemes for numerical solution of the Black-Scholz fractional equation, and in [13] the problem of the price of European options with an exponential maturity function is reduced to the calculation of multidimensional integrals. They are also the subject of [14], but already used in Bayesian statistics and for the development of probabilistic models of migration flows. Another such application is presented in [15], where the methods for calculating multidimensional integrals are used in the sensitivity analysis of the model for the study of air pollution. Finally, the works [16, 17, 18, 19, 23] are related in one way or another to the Monte Carlo methods and in particular: greater rate of convergence in the numerical integration of non-proprietary integrals using Sobol series, and in the case of the migration forecasting model, it is shown that the most effective method for this model is the one using the most important choice; the numerical results of quasi-Monte Carlo methods for calculating Sobol indices are analyzed; it is shown that in numerical calculation of multidimensional integrals of large dimension (over twenty), the optimal Monte Carlo method is more efficient than quasi-Monte Carlo methods, which use generalized Fibonacci numbers and Sobol series.

The candidate could submit official reports on the presence or absence of plagiarism concerning the works participating in the competition. The possible permissible percentage given with the help of specialized software on this issue would give an unambiguous and indisputable answer to the question. Based on his personal acquaintance with the candidate, the reviewer can describe him as a person who would not have thought of engaging in such an activity. But since this statement is subjective, the objective proof of lack of plagiarism is the place and manner of publishing the works submitted for participation in the competition - in refereed publications, the presence of impact factor, etc., which is a guarantee of lack of plagiarism and their high quality.

Of course, the division of labor and contributions in different directions is relative; some of them can be assigned to more than one of the categories, and all of them relate to a more general topic, which is actually the topic of the competition - methods for approximation of fractional derivatives, as well as some of their applications.

Dr. Yuri Dimitrov has a total of 47 citations (excluding self-citations). There are 39 citing publications that have an impact factor! Furthermore, the values of this impact factor in citations are on average around 2, with some exceeding 3 - for example with numbers [23] (IF 3.092) and [35] (IF 3.63) according to the numbering in the list of citations, submitted by the candidate.

The following citation publication cannot be overlooked: (for completeness, its bibliography is given in plain form: O Nikan, JAT Machado, A Golbabai, Numerical solution of time-fractional fourth-order reaction-diffusion model arising in composite environments, Applied Mathematical Modelling 89(1), (2021), pp. 819-836), which is numbered [37] with an impact factor 5.129.

It is noteworthy that these significant citations are from the last two years, which shows the relevance of the topic, the degree of achievement of the candidate and the application of the obtained results in international meaning.

Of the publications submitted for participation in the competition, 6 are independent, 4 have one co-author (in all of which the candidate is the first author). In the articles with more co-authors the candidate is the first author of 3 of them. There are no separation protocols for the individual contribution of publications with more co-authors, so the reviewer assumes that all have equal participation, but in those in which the candidate is the first author, his leading participation is assumed.

4. Critical remarks and recommendations of the reviewer.

I have no critical remarks. The materials of the competition are presented in a well-formed form. I would recommend the candidate to activate his publications in impact factor magazines. And another recommendation - when designing his publications better to clarify the need for relevant consideration, the physical meaning of their results and achievements and to emphasize their scientific, scientific and applied contributions and especially their importance in real aspects of reality.

5. Personal impressions of the reviewer about the candidate.

I know Dr. Yuri Dimitrov from his procedure for chief assistant at UF, and after that we had several meetings at Sofia University, Faculty of Physics, and other non-professional conversations. Based on this, I can say that he is extremely professional in his work (especially emphasizing his presentation of his procedure for chief assistant). He is always serious and responsible in performing his tasks at the Faculty of Physics at Sofia University. As a person he is very modest, attentive and polite, which contributes to the ability to work well in a team. These impressions confirm my opinion that the candidate fully satisfies the requirements necessary for holding the academic position of "Associate Professor" at UF.

CONCLUSION

Summing up, Dr. Yuri Dimitrov is a scientist with impressive scientific and applied contributions, which are entirely in the field of the competition. He has teaching experience with

students, which proves his pedagogical skills. His professional qualification fully corresponds to the topic of the announced competition - "Methods for approximation of fractional derivatives", respectively to the needs of the department that announced it.

Based on all that has been said so far and taking into account that the requirements of the law (ZRASRB) and the rules of UF are met,

I CONVINCELY RECOMMEND to the esteemed jury and the Faculty Council of the Faculty of Forestry of UF to elect Ch. Assistant Professor Dr. Yuri Mitkov Dimitrov to take the academic position of "Associate Professor" in the professional field 4.5. "Mathematics (Mathematical Analysis)" at the Faculty of Forestry, UF.

Sofia, April 5, 2022

Reviewer's signature: 

/George Boiadjiev/